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GEOMETRICAL NOTE-BOOK

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CONTAINING

EASY PROBLEMS IN GEOMETRICAL DRAWING

PREPARATORY TO THE STUDY OF GEOMETRY

For the Use of Schools

BY

F. E. KITCHENER, M.A.

LATE FELLOW OF TRINITY COLLEGE, CAMBRIDGE, AND MATHEMATICAL MASTER AT RUGBY.

SECOND EDITION, REVISED.

London:
MACMILLAN AND CO.
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PREFACE.

BEGINNERS in Geometry are met with two main difficulties, the one of grasping Geometrical ideas, and the other of seeing the force of Geometrical reasoning. These two giant difficulties are usually attacked together, and many boys are so encumbered with the double combat that they do not slay either of their giants. It is the object of this book to make some way in overcoming the difficulties of Geometrical conception, before the mind is called upon to attack Geometrical theorems. It is a safe guide in all teaching to make your pupils familiar with things before you give your theories.

Accordingly a few simple methods of construction are given, which, if in some cases rather long, at any rate satisfy the eye of the learner. By the graphical application of these methods to problems, it is thought that the ideas of the determination of points by the intersection of lines, of comparison of triangles, and of the indirect measurement of lines, will be gradually developed.

It is not intended that this book should do more than exercise the learner for some few months before he commences Geometry, either in the shape of Euclid or in some more modern form.

Before commencing the figures, the learner must possess ruler, compasses, and measure; and he will soon require a protractor. A pocketcase of instruments containing every thing that is wanted can be obtained for 4*s.* 6*d.* (Rowney and Co., Rathbone Place, London).

It is suggested that the learner should always draw the figure first on paper before he attempts to enter it into this book, in the space provided for it.

RUGBY, *April* 27, 1868.

The use of this Note-book at Rugby has shewn that problems of an extremely simple kind are required by beginners; accordingly in the Second Edition of this Note-book the number of easier problems has been much increased.

In compliance with the wishes of several Geometrical teachers, the use of the protractor has been altogether excluded from the first part.

A third part has been added containing a few of the simplest constructions connected with tangency and the circumscription of figures; this part might have been largely extended, but it is thought that the necessary training, preparatory to the study of Geometry proper, is better given by ringing the changes on a few constructions, than by burdening the memory with a large variety.

RUGBY, *Feb.* 23, 1872.

GEOMETRICAL NOTE-BOOK.

FIRST PART.

In the following constructions no instruments are allowed but the ruler and compasses, but the compasses may be used not only for the description of circles, but also for the transference of distances. In the numerical examples the scale may be used for the measurement of lines, but it is recommended that the protractor should not be used.

ANY common object may be measured in three directions; these measurements are called its *length*, *breadth*, and *thickness*.

The longest measurement of the three is generally called the *length*, and the shortest the *thickness*, but this depends on the position and use of the object you are measuring.

Which would you call the length, breadth, and thickness of this book? Measure them roughly with your scale, and enter your answer here. Length, Breadth, Thickness,

The measurement of an object straight upwards from the ground is often called its *height;* the same measurement, if measured downwards, is called its *depth.*

What measurements would you take to describe the size of the room in which you are sitting?

When an object varies in its measurements, you take its greatest length, its greatest breadth, and its greatest depth.

What measurements would you take to describe the size of a pencil, a ball, the table on which you are writing, a penny, a picture, a picture-frame?

Objects which have length, breadth, and depth, are called *solids*[1], and the outsides or bounding faces of solids are *surfaces.*

Hence surfaces have no depth; accordingly they have only two measurements, length and breadth.

Some surfaces are *flat*, as the paper on which this is printed, or the top of the table, or the floor: others are not, as the back of a cricket-bat, the outside of a ball, the surface of the earth.

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[1] It is recommended that young children should be made familiar with the appearance of some of the commoner solids. Boxes of Figures and Solids are sold by Myers and Co., 15, Berners Street, Oxford Street, W. Children cannot be too early made familiar with these; the schoolmaster will however find it necessary to supply what the mother has failed to teach.

Mention a few more surfaces that are flat or nearly so, and some that are not.

How would a workman test for himself whether a wall was flat or not? By taking a ruler, and seeing whether it can be placed anywhere against the wall without touching it all along; if it can, the wall has not a flat surface.

The figures you will have to draw will be drawn on flat surfaces, but figures may be drawn on surfaces not flat, as the figures on the surfaces of a celestial globe.

The edges of a surface are *lines;* lines have only one measurement, viz. *length.*

Lines are divided into those which are straight and those which are curved. Thus the line along which you look with one eye at an object is straight; again the line in which a ball is thrown is straight at first, but it immediately becomes curved.

Think of a few lines which are straight, and a few which are not.

How would you test whether a line was straight? Say a walking-stick or a fishing-rod. You would shut one eye, and make your other eye look from one end of the line to the other end, and see whether you could look all along the line at the same moment. If you could not, you would know the line was not straight.

The ends of a line are called points: you can cut a line anywhere, and make two lines of it; wherever you cut it you will get a point.

In how many points would one straight line cut another? In how many points would a straight line cut a ring?

A point has no depth, no breadth, no length; it merely marks position, as for instance, where two lines cut, or where one line ends.

By what measures do you measure lines, surfaces, solids? Learn them from your arithmetic.

Use of the Instruments.

The first thing which you have to learn is how to use your instruments. You will find you have instruments for drawing lines, and circles, and also for measuring lines.

Lines.

First learn how to use your ruler; its object is to enable you to draw a straight line between any two points; the line so drawn will be straight because it will follow the direction of the ruler, which is itself straight. If you doubt the straightness of your ruler you must apply the test given you above. A round ruler will be found more convenient than a flat one for drawing lines in ink. You will probably find among your instruments an instrument which will make finer lines in ink than a pen. You will notice that it has a small screw by which two points can be made nearer or farther from one another; the screw is intended to enable you to draw a thick line or a fine line, according to your wants.

A ruler is also used for *lengthening* or *producing* a straight line in either direction.

1. Draw 6 lines of different lengths on the first empty page, some in ink, some in pencil, taking care to make them very fine, and yet quite distinct. Then measure them, and record their lengths in inches.

To do this you will have had to use your measure (probably six inches long), which you will also find useful sometimes as a ruler; but you will have found it rather hard to measure exactly those lines whose length cannot be expressed as a whole number of inches.

Now learn to use your compasses for the more exact measurement of lines.

You will see that the compass has two pointed legs; stretch the compass till the ends of the legs are the same distance apart as the length of the line you wish to measure.

Then apply the compass to your measure, and you will be able to tell the length of the line very accurately.

You will see that one of the inches is subdivided into 12ths, and let us suppose that you find the line you wish to measure is between 2 and 3 inches. Then place the further leg of the compass upon the figure 2 in the inch scale, and then the other leg will fall somewhere in the inch that is divided into 12ths, you will thus be able to tell the length of the line correctly to twelfths of an inch. If this is not sufficiently accurate, you can use the half or quarter of an inch scale, with which, each unit being divided into 12 parts, a line may be measured accurately to the 24th and 48th parts of an inch[1].

2. Draw a three-sided figure with unequal straight sides, and measure them accurately.

3. Draw lines $2\frac{5}{12}$ in., $3\frac{2}{3}$ in., $\frac{3}{8}$ in. long respectively, taking the lengths from your scale.

4. Draw any four-sided figure with two sides equal, and the other sides unequal, making the equal sides 2 inches long, and measure the other two sides.

5. Draw any unequal-sided three-sided figure, and *produce* each side two inches.

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[1] It is not thought necessary here to give a description of the use of the diagonal scale: the teacher will of course at his discretion instruct any of his pupils in its use, but for beginners sufficient accuracy may be obtained without its use.

Circles.

You have now learnt to make, to produce, and to measure straight lines. You will find that your compasses will also enable you to make *circles.*

A *circle* is a figure drawn on a flat surface bounded by one line (called its *circumference*), such that all points in the circumference are the same distance from a certain point within it called its *centre.*

The distance between the circumference and the centre is called the *radius;* a line through the centre terminated each way in the circumference is called the *diameter.*

The *compasses* are used to make circles; one leg of the compasses is put at the point which is to be the centre, and the other leg is made to trace out the circle.

You must be careful to see (1) that the pointed leg of the compass is replaced by that which ends in a pen or pencil, and (2), if you want to make a circle of a particular *radius,* that the distance between the points is the exact distance you require for radius.

In using the pencil you will often find it convenient to bend one leg of the compass, which has a joint for that purpose.

6. Describe circles of (1) one inch radius, (2) two inches radius, (3) half-inch radius, (4) three inches diameter.

7. Describe two equal circles radius 1½ inches, so that the circumference of each may pass through the centre of the other.

8. Describe three unequal circles so that the centre of each may lie on the circumference of one of the others.

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9. *First Construction.* How to bisect any given line, that is, to divide it into two equal parts by finding its middle point.

Take any line and bisect it by the following method.

Suppose your line to be called *AB*, *A* being one end, and *B* the other; you are required to bisect it.

First describe a circle round *A* as centre; you must be careful to make the radius of this circle more than half the distance from *A* to *B*.

Secondly, round *B* as centre describe another circle of equal radius to the last.

These two circles will cut one another in two points; call them *C* and *D*; draw a line from *C* to *D*; this line will cut *AB* in the middle point. Call this point *X*.

AB will now be bisected in *X*.

Notice. In the constructions where no figure is given, the pupil is expected to draw it for himself from the directions given; if the figure is given, the pupil is to copy it in the order of the directions given.

10. Transfer from your scale with your compasses a line two and a half inches long, and divide it into halves by *First Construction.*

11. Take a line three inches long, and divide it into quarters by bisecting it, and then again bisecting each half.

12. Take any threesided figure or *triangle* with three unequal sides; bisect all three sides, and produce the lines, by which you have bisected the sides, till they meet. If the figure is correctly drawn, the three lines will be found to go through the same point.

13. A fly is 1 inch from the centre of a given circle 3 inches in radius; another fly is stationed half-way between the first fly and centre of the circle; let the first fly make for any point you like in the circumference, and find how far he will be from the second fly when he has got half-way.

Ans. $1\frac{1}{2}$ inch.

14. Draw any *quadrilateral*, or four-sided figure with unequal sides, and describe semicircles on the sides as diameters. [To find the centre of each semicircle you must bisect each side.]

Right angles and perpendiculars.

If a line have one end fixed, and revolve about it, the ***direction*** of the line is always changing. Think of a clock-face, and the minute-hand revolving about the centre of the face; or think of the needle of a mariner's compass, which, after oscillating backwards and forwards, indicates the ***direction*** to the north.

When such a line has revolved completely round, it is said to have made one revolution; when it has not gone all round, the part of a revolution through which it has gone is called an ***angle.*** The angle is measured by the quantity of turning, through which the line has turned.

A whole revolution is divided into four right angles; hence when the line is pointing in exactly the opposite direction to what it did before, then it has gone through two right angles.

Mention some times in the day at which the clock-hands point in opposite directions.

If a man travelling north were to change his direction through two right angles, in what direction would he go? In what, if he turned to his left through 3 right angles? In what, if he turned to his right through 2½ right angles?

If the hand of a watch were to revolve about the centre from pointing to twelve till it pointed to six, it would have to go through two right angles; if it stops in any direction between these directions, the angle it has gone through and the angles it will have to go through before it point to six make up together half a revolution or two right angles.

If it stops half-way, as in pointing to three, when it has gone through a quarter of a revolution, it is said to be ***perpendicular*** or ***at right angles*** to the line joining twelve and six.

The *First Construction* may be used to erect a perpendicular at the middle point of a given line.

15. Take a line 5 inches long and erect a perpendicular at its middle point.

Right angles are subdivided into degrees, minutes and seconds. Learn the table of angular measurement from your arithmetic.

What angle is there between the hands of a watch at 6½, 8, 9¾, 11?

Ans. 15°, 120°, 22½°, 30°.

An angle will be described by stating (1) about what point the line is revolving, (2) in what direction the line was at first, (3) in what direction the line now points.

The lines marking the old and new directions are called the ***arms,*** and the point about which they revolve the ***vertex.***

Hence we can draw the angle if we know the vertex and one point on each of the arms.

16. Suppose you have any three points A, B, C given you: draw lines from A to B and from A to C; an angle will thus be formed at A, with A for vertex, and AB, and AC for arms; this angle is called the angle BAC; the letter representing the vertex is placed in the middle to indicate that the arms are drawn from A to B and from A to C. What new angles will be made by joining BC?

17. *Second Construction.* How to erect a line perpendicular to another, at a particular point in the line.

Take any line AB, and suppose it is required to erect a perpendicular at some point C in the line, which is *not* the middle of the line. First, with centre C and any radius not too large describe a circle cutting the line AB in the points D and E.

Secondly, describe two equal circles with centres D and E (as in the *First Construction*); let these circles cut each other in F. Join CF; CF is the required perpendicular.

18. Three men start from the same point, going N., S., E. at the rates of 3, 4, 5 miles an hour respectively; draw two lines at right angles by *Second Construction* so as to represent the N., S. and E. directions. Find by measurement how far they are from one another at the end of two hours. Scale $\frac{1}{4}$ in. = 1 mile.

Ans. 14, $11\frac{2}{3}$, $12\frac{5}{6}$ nearly.

19. Take a line three inches long, and erect perpendiculars at the distance of half an inch from each end: make the perpendiculars two inches high and join their extremities: thus making a *square.*

20. A path goes straight for 3 yards, then goes at right angles for 8, then goes straight to the middle point between where it first started and where it turned off. Find how long it is. Ans. $19\frac{1}{10}$ nearly.

21. *Third Construction.* How to drop a perpendicular from a given point above or below a line.

Let AB be the line, C a point anywhere above or below it from which we want to drop a perpendicular. Take C as centre, describe a circle so as to cut the line AB in the points D and E: any radius may be chosen which is not too short. With D and E as centres, and with any, the same, radius, more than half of DE, describe two circles so as to cut one another in F. Join CF, which will be the perpendicular, and produce it (if necessary) to meet AB.

22. Take any triangle (having unequal sides) and drop a perpendicular on the longest side from the opposite angle.

23. Find two points in a given line which shall be 1 inch from a point whose perpendicular distance from the line is $\frac{1}{2}$ an inch. How far are they apart? Ans. $1\frac{35}{48}$ inch.

24. *Fourth Construction.* To bisect any given angle.

Take any angle BAC, with A as vertex, and suppose you want to divide BAC into two equal parts.

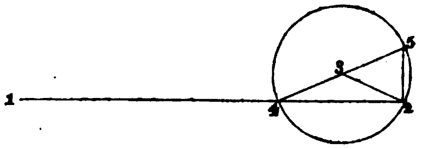
From centre A, with radius less than either AB or AC, describe a circle cutting them in points D and E. We want to find a point at the same distance from D and E which we can join with A. To find this point describe two circles, one from centre D and the other from centre E, using the same radius for both circles, which may be any length sufficient to make the circles cut. Let them cut in F, then join AF, and the angle will be bisected by AF, that is; the angle BAF will be half of the angle BAC.

25. Describe angles of 45° and 22½°. To find 45°, a right angle must first be found by *Second Construction*, and then bisected.

26. Take any triangle with unequal sides; bisect all three angles; the three *bisectors*, or bisecting lines, will be found to go through the same point.

27. Describe any quadrilateral with sides 4, 3, 1 and 2 inches successively, and bisect all four angles: the four bisectors will go through the same point, if the figure is drawn correctly.

28. *Fifth Construction.* How to erect a perpendicular at the end of a line by means of any circle through the end of the line.



Take 12 the line; let a perpendicular be required at 2. Take any point 3 not far from 2, outside the line 12 and towards 1.

Describe a circle with centre 3 so as to pass through the point 2, and let it cut the line a second time in the point 4.

Draw a line from 4 through the centre of the circle 3 till it cuts the circumference again in a point to be called 5.

The line joining 2 and 5 will be the perpendicular, and the angle at 2 a right angle.

This figure is to be copied, and the points are to be obtained in order, 1, 2, 3, then 4, and last of all 5.

29. Draw a line 2 inches long and at one extremity erect a perpendicular 1½ inches high.

30. Describe a *square* (i.e. a four-sided figure with all its sides equal and all its angles right angles), each side to be 2 inches, and describe a circle to pass through all four corners.

31. The height of a wall is 20 feet; required the length of a ladder which will reach to the top, when the foot of the ladder is 15 feet from the wall. Draw a figure and measure the ladder. Ans. 25 feet.

32. If the ladder were 20 feet, and its foot 12 feet from the wall, how high would the wall be? Ans. 16 feet.

33. A man walks two miles, then turns to his right-hand through a right angle for three miles, then turns to his left through a right angle and walks a mile. Draw a plan, assuming only an inch to represent a mile, and find how far he then is from his starting-point. Ans. 4¼ miles nearly.

34. A man walks 3 miles, turns through 45° to his left, goes half a mile, turns through 90° to his left, and goes 1 mile further. Shew that he is about 2⅞ miles from home.

Caution:—When it is said that a man turns through 45° to his left, it means that he turns through an angle of 45° from *the direction in which he would have gone, if he had gone on straight.* You should therefore produce your line beyond the point where he stops, and the new direction will make an angle of 45° with this line.

35. *Sixth Construction.* How to copy a triangle, by measuring the lengths of the three sides.

Take any triangle ABC: transfer with your compasses a line equal to the line BC and let the line so transferred be called EF.

From centre E with radius equal to AB describe a circle: then from centre F with radius equal to AC describe a circle: let these two circles meet in D: join DE and DF: then DEF will be an exact copy of ABC.

36. Suppose that you have taken the measurements of the sides of a triangular field, viz. 10 poles, 15 poles, and 20 poles, draw a plan of the field (scale 10 poles = 1 inch).

37. The direct distance from Coventry to Warwick is 10 miles, from Rugby to Coventry is 11½ miles, and from Rugby to Warwick 15 miles; how far is Rugby from the middle point between Coventry and Warwick? Ans. 12¼ nearly.

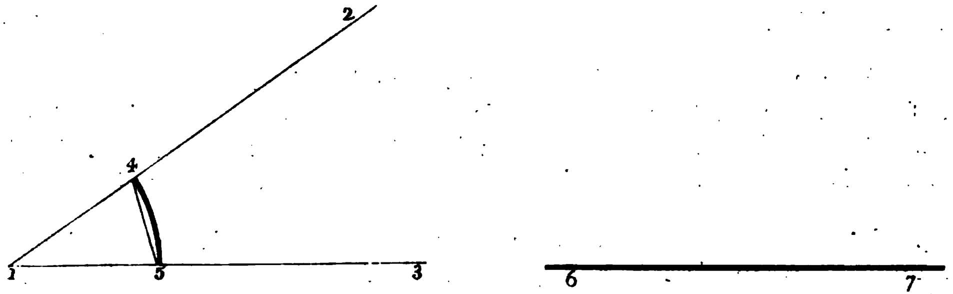
38. Describe a triangle which shall have its sides 1½ and 2 inches, and the angle between them a right angle. Ans. Third side = 2½ in.

39. Find a point 7 ft. distance from each of two points which are 8 ft. from one another, scale 4 feet = 1 inch.

40. *Seventh Construction.* To copy an angle already drawn.

Suppose the angle which is required to be copied be called 213, and suppose it is wanted to make an angle equal to it so as to have its vertex at 6, and with 67 for one of the arms.

From centre 1 with any radius describe a circle, cutting 12 and 13 in 4 and 5: join 45; then the curved line between 4 and 5 is called the *arc* or bow, and the straight line joining 4 and 5 is called the *chord* or bow-string.



With the point 6, where the angle is wanted, as centre, describe a circle with same radius as before, cutting the line 67, which is to be one of the arms of the required angle, in 8; then measure off a chord 89 equal to 45, and join 69; then 967 will be the angle required.

The pupil is to complete the above figure, and should notice that 6 *is not intended to be exactly at the end of the line.*

41. Describe a triangle with each of its sides equal to any line you choose to take.

Such a triangle is called an *equilateral* triangle.

42. Describe a triangle (by *Sixth Construction*) with any line as base, and each of the other sides double the base.

43. In this and several other examples it will be assumed that the three angles of any triangle are together equal to two right angles, or 180°[*], and that in an equilateral triangle described as in (41) all the angles are equal and each equal to 60°.

Describe an angle of 60° at the end of any line.

44. Construct an angle of 30°, using (43) and *Fourth Construction.*

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[*] In order to familiarize the pupils with this geometrical fact, the teacher may test its truth in the case of any particular triangle in the following way. Let any triangle be drawn on paper or on the black board, and let a pencil be laid along its base with its middle on one of the angular points; let the pupils be asked to notice in which direction the sharpened end is directed; let the pencil then be made to rotate through that angle: let the pencil then be transposed to the next angular point, its direction being unaltered; then let the pencil be made to rotate through the angle at that point in the same direction of rotation as before, and similarly through the third angle: the pencil will then be found to point in the opposite direction to that in which it pointed originally, and the pupils will admit that the angles of that particular triangle make up two right angles.

45. *Eighth Construction.* To make an angle equal to the sum of two given angles.

Take any two unequal angles at the points A, D, viz. BAC, EDF; at the point A with arm AB on the side remote from C construct an angle (by the preceding construction) equal to the angle EDF, with outer arm AG, then the angle GAC between the two outer arms will be equal to the sum of the two given angles.

46. *Ninth Construction.* To make a triangle, of which two sides and the angle to be contained by them are to be copied from two given lines, and one given angle.

Set yourself any two lines AB, CD to be the sides, and any angle E to be the given angle.

Lay down a line FG equal to AB, and then by the *Seventh Construction* make an angle at F equal to E, and let this angle be called GFH; then in the arm FH find a point K such that FK is equal to CD, and join GK. The triangle is complete.

47. *Tenth Construction.* To make a triangle of which one side and the two angles, of both of which the side is to be an arm, are to be copied from one given line, and two given angles.

Set yourself any line AB to be the base of the triangle, and take C and D any two angles for the given angles.

Lay down by transference with your compasses a line EF equal to AB, and at E copy (by *Seventh Construction*) the angle C, and again at F copy the angle D, making EF an arm of each of these angles, and produce the other arms till they meet in G.

Then EGF is the triangle.

48. *Eleventh Construction.* To make a triangle of which one side and the two angles are to be copied, one of the angles to be opposite to the given side.

Suppose any line AB to be given as the required side, and any two angles D, and E, of which E is to be opposite to the side AB.

Lay down by transference a line FG equal to AB: at F by the *Seventh Construction* make an angle equal to the angle D, and lengthen FG to any point H and at G from the produced line GH lay off an angle equal to the sum of D and E (by the *Eighth Construction*); produce the two new arms from F, G till they meet; the point where they meet (say O) will be the third point of the triangle; the angle at O will be found on measurement to be equal to the angle E.

49. Suppose your notes of a triangular field to be; base $AB = 100$ yds., angle at A 30°, angle at B 45°; draw a plan of the field by the *Tenth Construction*, and measure the other side ending in A. Ans. 74 yds. nearly

50. Suppose your rough notes of a triangular field ABC to be $AB = 20$ chains, $AC = 15$ chains, angle at $A = 60°$, draw a plan of the field by the *Ninth Construction*, and find the length of BC. Ans. 18 chains nearly.

51. Draw a plan of a triangle whose sides are 11 and 23 chains and included angle 45°, and shew that the third side is nearly $17\frac{1}{8}$.

52. Suppose in *Construction XI.* the base to be 2 inches, the angle adjacent to it 30°, and the angle opposite to it 45°, and describe the triangle as directed.

53. A man walks 2 miles, turns through an angle of 60°, walks 2 miles, turns the same way through the same angle and walks 2 miles. Find how far he is from where he started. Ans. 4 miles.

Caution:—Here again (see 34) the angle of 60° is to be laid off from the direction in which he would have gone if he had not turned off.

54. *A* and *B* are two forts with a river between them; a man goes from *A* to a bridge, *C*, 10 miles off: then goes to *B* from the bridge, 12 miles, by a road making an angle of 30° with the road from the bridge to *A*. Draw a plan, and find how far *A* is from *B*. *Ans.* 6 miles.

55. From any point *O* in the circumference of a circle, draw three chords to any three points in the circumference; bisect these chords and on each of them as diameter draw three circles (each of them of course passing through *O*); these three circles will intersect in three points (not counting *O*), which if the figure is drawn correctly will be in the same straight line.

For the meaning of the word *chord* see (40).

56. Take any triangle *ABC*, and describe squares on the sides *BA*, *CA*, viz. *BFGA* on *BA*, and *CKHA* on *CA*; drop a perpendicular from *A* on *BC* (by *Third Construction*, page 19), and join *FC*, *KB*: if the figure is drawn correctly, *FC* and *KB* will intersect on the perpendicular.

57. Draw a line *CD* obliquely to *ACB*; bisect the angles *ACD*, *BCD*; and shew that the bisectors are at right angles to one another.

58. Describe six equilateral triangles round a point, not overlapping one another, and having a common vertex at that point. These six triangles will make a regular hexagon.

59. A man 100 ft. from the base of a tower finds that the tower is opposite to (or *subtends*) an angle of 30° at the ground. Draw a plan, and measure the tower. Ans. $57\frac{3}{4}$ ft.

60. A man wishes to find the distance of a ship at sea: he measures 200 yards on the beach, and, taking that line as base *AB*, and the ship as the point *C*, he finds the angle $CAB = 90°$, the angle $CBA = 60°$. Shew that the ship is 346 yards from *A*.

61. *Twelfth Construction.* How to draw a line parallel to another at a given distance from it.

Straight lines which do not meet, however far they are produced in either direction, are called *parallel.*

Suppose it to be required to draw a line parallel to any other line *AB*, and at an assigned distance from it.

Erect perpendiculars at any two points in *AB*; make their length that of the given distance; join their ends, and you will have a line parallel to the given line, which may be produced either way.

62. *Thirteenth Construction.* How to draw a line parallel to a given line through a given point not in the line.

Take any line *BC*, and any point *A* not in the line *BC*, and suppose that it is required to draw a line through *A* parallel to *BC*.

With centre *A* describe a part of a circle cutting *BC* in *D*: then with centre *D* and radius *DA* describe a part of a circle to cut *BC* in *E*: measure the distance *EA* with your compasses, and by transference take an equal chord in the other circle to be called *DG*: this chord is to be measured from *D* on the same side of *BC* as the other chord: join *AG*, and produce it—this line will be the parallel.

63. A man finds that a tower subtends an angle of 30° at his eye; he approaches the tower 27 yards, and finds that the angle is doubled. Find the height of the tower. Ans. $23\frac{7}{20}$ nearly.

64. Describe an equilateral octagon or eight-sided figure in a circle whose radius is 1 inch.

65. Bisect a line without compasses, by the following rule. Let *AB* be the line. Draw a line *AP* from *A*, making any angle with *AB*, and along it mark off three equal lines, viz. *AC*, *CD*, *DE*, join *EB*, and produce it to *F*, making $BF = EB$. Join *CF*, which will cut *AB* in its middle point *O*.

Test your result by *First Construction.*

66. *Fourteenth Construction.* To divide a given line into any number of equal parts.

Take a given line, *AB*; suppose you want to divide it into three equal parts. Take another line, *AP*, making any angle with *AB*, and measure off from *A* *equal* parts of any convenient length along *AP*; these parts should be one more in number than the number of parts into which *AB* is to be divided, and let us suppose them to be called *AC, CD, DE, EF*: join *FB* and produce it to *H*, making *BH* equal to *BF*; then draw a line from *H* to the second point from the end of the equal parts, (in this case *D* which is the second letter from *F*); let this line *HD* cut *AB* in *O*; *OB* will be the required fraction of *AB* (in this case one-third), and other parts equal to it can now be cut off.

67. Divide a line of 3 inches long into 5 equal parts; and a line of 5 inches long into 11 equal parts.

68. Remembering the meanings of arc and chord (see *Seventh Construction*), find the chord of 45°, when the radius is 2 inches.

Ans. Rather above $1\frac{1}{2}$ inches.

What line would be the sagitta or arrow to the bow-string? Draw it, and measure it. Ans. $\frac{1}{8}$ in. nearly.

69. Describe in a given circle an eight-sided figure which shall have its sides subtending alternately angles of 60° and 30° at the centre.

70. Describe an arc whose chord is equal to the radius of the circle (2 inches); bisect the arc, and measure the chords of the semi-arcs. Ans. $1\frac{1}{24}$ in.

71. Draw a picture of the compass, marking the N., W., S.W., and S.W. by W. points. What fraction of a right angle is there between the S.W. by W. and N.W. by N. points? Two men start together, going one N.W. at 40 miles an hour, the other S.W. by S. at 30 miles an hour; how far apart will they be after ten hours? Ans. 90°; 588 miles nearly.

72. A man walks three miles, and then walks in a circle through whose centre he has passed and whose radius is one mile, till he has made the chord of the circle equal to its radius. Find how far he is then from home.

Ans. $2\frac{5}{8}$ nearly.

73. Describe a four-sided figure which shall have two sides parallel and one inch apart, and the other two sides equal, but not parallel, and two inches long.

74. Shew by a figure that you cannot draw a triangle with sides 1, 2, and 3 inches.

75. Draw a cross vertical section of a ditch whose depth is 3 feet, breadth at the bottom 2 feet, and at the top 4 feet, the sides being equal; and measure the side. Ans. side = $3\frac{1}{6}$ft. nearly.

76. Describe a square with side one inch, and on the four sides describe equilateral triangles; join the vertices, and shew by measurement that the figure so formed is a square.

77. A man measures a four-sided field; he finds that the distances across it from the opposite corners are 100 and 120 yards; that these two distances bisect one another in the middle, and are inclined at an angle of 30° to one another. Find the length of the shorter side. Ans. $30\frac{1}{18}$.

78. Try to draw a triangle which shall have one side, *AB*, 10 feet, the angle *ABC* 30°, and the side *AC* 7 feet. To determine the point *C* it will be necessary to describe a circle with centre *A*, and radius 7 feet. This will cut *BC* in two points (about $9\frac{1}{2}$ feet apart), either of which will suit for the third point of the triangle.

79. A man 50 yards from the edge of a moat close round a castle finds that the castle tower subtends an angle of 45°, while at the edge of the moat it subtends 60°. Find the width of the moat. Ans. 68 yds.

80. Describe an equilateral triangle (base 2 inches), bisect one of the angles, and place another equilateral triangle symmetrically so as to make a six-pointed star, and shew by measurement that each side of each triangle is trisected.

81. Three ships *A*, *B*, *C*, start together from a port, *A* going south, *B* south-west, and *C* west: and their rates are such that they keep in a straight line with one another. If *A*'s rate is 10, and *C*'s 6 miles an hour, find *B*'s. Ans. $5\frac{1}{3}$ miles.

SECOND PART.

The use of the Protractor, and Miscellaneous Exercises in previous Constructions.

In any of the following exercises, the protractor may be used for laying off and measuring angles; directions will now be given for its use.

A protractor is used (1) to lay off an angle of a given size (say of 70°) at a given point in a given line, and (2) to measure an angle already drawn.

(1) To lay off an angle of a given size.

Make the plain or lower edge of the protractor coincide with the *line from which* the angle is to be set off: the stroke in the middle of this edge is to coincide with the *point at which* the angle is to be set off; you must then make a mark on your paper, at the point which coincides with the stroke on the protractor marked with the number of degrees in the angle required to be drawn; then take away the protractor, and draw a line through this new point and the point in the given line at which the angle was to be set off.

(2) To measure an angle already drawn.

Place the protractor as before, and note at which stroke on the protractor the line containing the angle cuts the edge of the protractor: the figure there marked will give the number of degrees in the angle.

In laying off and measuring angles with the protractor you will find that you sometimes want to lay off the angle in the opposite direction to that in which the hands of a watch rotate, and sometimes in the same direction.

To enable you to lay off in either direction you are given two rows of figures; one graduated from 0° to 180° in the same direction as the hands of a watch; this is the *outer* of the two rows of figures; the other graduated in the opposite direction: this is the *inner* of the two rows. Should you make a mistake and use the wrong one, the angle will come out markedly too large or too small, so that you ought to see your mistake at once.

1. Take any line, and at one end of it lay off, with your protractor, angles of 70°, 50°, 170°, 110°.

2. Draw two lines forming an angle of 40°, and bisect this angle.

3. Draw any triangle, and measure the angles with your protractor; see whether the sum of the angles comes to 180°.

4. Measure also the angles in the quadrilateral drawn in Question 4, Part I. Shew, by measurement, that the sum of the angles comes to 360°.

5. Describe a triangle with sides 3, 4, and 5 half-inches, and measure the angles. Ans. 90°, 37°, 53°, nearly.

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6. A tower stands on a rock; a man from the sea at 100 yards from the foot of the rock finds the angle of elevation of the *foot* of the tower 15°; he then rows 100 yards further off and finds the elevation of the *top* of the tower 15°. Find the heights of the rock and of the tower. Ans. Both about 27¼.

7. The angle of *elevation* of an object above an observer means the angle between a horizontal or level line in a direction towards the object and a line elevated upwards and passing through the object.

So the angle of *depression* of an object below the observer means the angle between a horizontal line towards the object and a line depressed downwards and passing through the object.

Find the height of a tower whose angle of elevation 200 yards off is 15°.

Ans. 54½ yards.

8. At the top of a mountain, the angle of depression of the peak of a neighbouring mountain is 5°: if the difference of the heights of the two peaks be known to be 500 feet, find the distance from the centre of the base of one mountain to that of the other. Scale ½ in. = 500 feet.

Ans. 1900 yds. about.

9. Describe a triangle with base 2 inches, and angles at the base 70° and 30°. Measure the third angle, which will be found to be 80°; and describe another triangle with base 1 inch, and the same angles, and shew that the sides of this triangle are halves of those of the first triangle.

10. Pairs of triangles which have their angles equal each to each are of the same shape, and are called *similar:* they are the same triangles on different scales.

Draw a triangle whose sides are 1, 1½, 2 inches, and a triangle whose largest side is 1½ inches with angles copied from the former triangle; and satisfy yourself by measurement that any side of the new triangle is ¾ of the corresponding side in the old triangle.

11. A man wishing to find the distance of an enemy's fort measures a base of 100 yards, and finds that with the fort as the vertex or third point of a triangle, and the 100 yards as base, the angles at the base are each 67½°. Find how far the fort is from either end of the base; and measure the third angle.

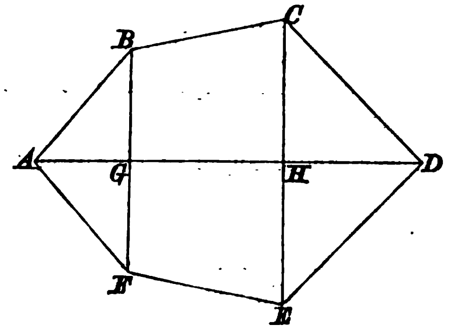
Ans. 130 yds.; 45°.

12. Find the distance between two inaccessible objects, which can only be seen together from one point, from the following notes:

Let 1, 2 be the objects, 3 the point from which both can be seen. The angle 435 is 135°. I measure 34 and 35, each 100 yards; and observe the angles 341 and 134, and find them 30° and 80°. I measure the angles 235 and 352, each 67½°. Ans. nearly 80.

13. Make a plan of a six-sided field from the following notes. I walk from *A* to *D*, 200 yards; when I am at *G*, 50 yards from *A*, *BG* is perpendicular to *AD* and equal to 60 yards; when I am at *H*, 70 yards from *D*, *HC* is perpendicular to *AD* and equal to 80 yards: *E* and *F* are similarly placed on the other side of *AD*. Find how far the field is round. Ans. 514 yds.

The notes are explained by the accompanying incorrect figure.



14. Draw a figure for the same problem as (12), the measurements being 34 = 50 yds.; 35 = 200 yds.; the angles 341, 134 = 28°, and 78°; the angles 235, 352 = 60°, and 80°; and 435 being a straight line. Ans. 288 yds.

15. The distance between wickets is 22 yds.; Long-on and Long-off are on opposite sides of the line between wickets; the line joining either of them to the bowler's wicket is 22 yds. long, and subtends an angle of 22½° at the batter's wicket; how far are they from one another? Coverpoint is the third point of a triangle whose base is the line of wickets, and whose base angles are 75° at the batter's and 60° at the bowler's wicket. Find how far he is from Point, who is on the same side of line of wickets, distant 8 yards from the batter's wicket, in a direction perpendicular to the line between the wickets.

Ans. 31 yds; 18 yds. nearly.

16. Draw a rectilineal figure *ABCD* of which the sides *AB* and *AD* are each equal to 2 inches, and inclose an angle of 25°, and the sides *BC* and *CD* each equal to 1 inch, and measure the angle at *C*. Ans. 51⅛°.

17. Describe a circle 1½ inches radius, and in it place a straight line 2 inches long, with extremities in the circumference.

18. Construct a six-sided figure *ABCDEF*, having its corners in the circumference of a circle of 2 inches radius, centre *O*. The angle $AOB = 60°$, $BOC = 70°$, $COD = 50°$, $DOE = 80°$, $EOF = 50°$. Measure the angle *FOA*, and state which of the radii from *O* are parts of the same diameters. Ans. 50°.

19. Draw a straight line 4½ inches long and divide it by *Fourteenth Construction* into 5 equal parts.

20. Construct (as in 78 of Part I.) a triangle having one of its sides AB equal to 3 inches, another AC equal to $3\frac{1}{4}$ inches, and the angle at B equal to 50°, and measure the remaining angles. Ans. 45°, 85°.

21. Construct a triangle the lengths of whose sides are $2\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{3}{4}$ inches respectively, and measure the angles. Ans. 51°, 59°, 70° nearly.

22. A see-saw, 10 feet long, rests on a pivot 4 feet from the ground, and starts from a horizontal position, through what angle will it turn before one end touches the ground; and how far from the ground will the other end be?

Ans. $53\frac{1}{8}$°; 8 feet.

23. In a circle, radius 2 inches, describe a twelve-sided figure whose sides subtend angles of alternately 15°, and 45° at the centre.

24. Construct an equilateral triangle of height 1 inch, and measure the side (using the fact that the perpendicular from the vertex bisects the vertical angle). Ans. $1\frac{1}{8}$ in.

25. Construct a triangle, with base one inch, and each side double the base, and measure the vertical angle. Ans. nearly 29°.

A triangle with two equal sides is called *isosceles*.

26. Construct a square whose diameter shall be two inches.

27. Draw an equilateral triangle of 3 inches side, one corner of it to be made the centre of two squares of 2 inches diameter, having their sides parallel and perpendicular to the sides of the triangle.

28. Divide a line $3\frac{1}{2}$ inches long into two parts so that one part may be a sixth of the other.

29. Construct an isosceles triangle, base $1\frac{1}{2}$ inches, the angles at the base being double that at the vertex. (Remember all three angles $= 180°$.)

30. Draw a plan of two goal posts 18 feet high, and 24 feet apart, with a cross bar 10 feet from the ground.

31. Draw a ground plan, finding a spot whose distance from the goal line is 12 yards, and whose distance from the further goal post is 20 yards, and measure its distance from the nearer goal post. Ans. $14\frac{5}{12}$ yds.

32. Find at what angle of elevation a ball must be kicked from the spot above mentioned so as just to hit the nearest point of the cross bar.

Ans. $13\frac{1}{4}$°.

33. $AB = 1$; $BC = 1.2$; $CD = 1.4$; $DE = 1.6$; $EF = 1.8$; angle $ABC = 140°$; $BCD = 130°$; $CDE = 120°$; $DEF = 110°$. Construct a polygon $ABCDEF$ with these data, and measure the side FA; scale 1 inch $= .8$; and show that BC is parallel to FE.

Ans. A little more than 2.

34. A man walks three miles, turns to his left through an angle of 45°, goes two more miles, turns through 60° in the same direction goes one mile; and then, after going one more mile in the same direction in which he started, goes straight home; how far has he been in all?

Ans. 12½ miles nearly.

35. Describe an equilateral duodecagon or twelve-sided figure in a circle of two inch radius, and on one of the sides of the duodecagon describe a hexagon (see 58 in Part I.).

36. A man walks 100 yards from a tower, and then finds the tower is opposite to an angle of 20° at that distance, find height of tower. Ans. 36⅓ yds.

37. Two roads branch off at A, making an angle of 45° with each other. Find the distance of two places, one being 7 miles from A along one road, and the other 8 miles from A along the other. Ans. 5¾ miles.

38. If one angle of a triangle be 20° greater than the second, and the third be a right angle, find the second. And draw the triangle, the side opposite the right angle being 3 inches, and measure the shortest side.

Ans. 35°; 1¾ nearly.

39. A tower 50 ft. high stands on the edge of a cliff, and the angle of depression of a boat as seen from the top of the tower is 20°, and from the bottom 14°. Determine the height of the cliff and the distance of the boat from its foot. Ans. 36 yds., 145 yds.

40. Divide a line two inches long into seven equal parts.

41. Describe an isosceles triangle, having a base ¾ inch long, and vertical angle of 40 degrees.

42. A man walks for 3 miles straight; he then turns through an angle of 30° and proceeds for one mile; he next resumes a direction *parallel* but *opposite* to his first, and again walks 3 miles.

Draw a map of his route on a scale of one inch to a mile, and find how far he is from his starting point. Ans. 1 mile.

43. A man measures AB one side of a triangular field, and finds it to be 15 poles: he measures the angle at B which he finds to be 70°; he walks from B to C, but neglects to measure it. He, however, finds the angle at C to be 45°. Draw a plan of the field, and find how long BC is. Ans. 20 poles.

44. Draw two lines making an angle of 40°; and find a point which shall be an inch distant (perpendicularly) from one of the lines, and 1½ inches from the other.

8

THIRD PART.

This part contains a few of the simplest constructions relating to the circle, and the figures should be drawn with the same instruments as in the First Part, and without the Protractor. A knowledge of the Second Part is not required for the Third Part.

1. To find the centre of a given arc of a circle.

Suppose any arc given you, whose centre you are required to find.

Construction. Take any two chords of the arc (that is, lines whose ends lie in the circumference of the arc); erect perpendiculars at their middle points (by *First Construction*, Part I.); the intersection of these perpendiculars will be the centre.

2. To describe a circle through any three points not in the same straight line.

Construction. Take any three points; join them by straight lines so as to make a triangle; then erect perpendiculars at the middle points of two sides of the triangle, and their point of intersection will be found to be the centre of a circle which passes through all three points.

3. Describe (by *Sixth Construction*, p. 25) a triangle with sides 5, 6, 7 quarter-inches, and describe a circle to pass through its angular points by the Construction in (2).

4. Construct a square of two inches side, and about it describe a circle.

Notice. A circle is said to be *described about* a straight-sided figure when it passes through all its angular points.

5. Construct a trefoil by describing an equilateral triangle, bisecting each side, and describing three circles, with the angular points as centres, and radius equal to half the side of the triangle.

Caution: the portions of the circles which would fall inside the angle need not be drawn.

6. To draw a line touching a given circle at any given point in the circumference.

Notice. A line touching a circle is called a *tangent*, and is always perpendicular to the radius at the point where it touches.

Construction. Join the point to the centre, and erect a perpendicular at the given point by *Second* or *Fifth* Constructions (Part I.).

7. Draw a line 3 inches long, and draw a circle 1 inch in radius to touch it at its extremity.

8. To draw a tangent (or touching line) to a given circle from any point outside the circle.

Construction. Suppose O to be the centre of the circle and A the point outside it; join OA, bisect it, and on it as diameter describe a circle, which will cut the original circle in two points, call them B and C.

Then OB and OC will both be tangents to the circle.

9. Draw a tangent to a circle two inches in radius from a point three inches from its centre.

10. A man stands on the top of a globe 15 feet in radius; supposing his eye to be 5 feet from the ground, how far from his eye is the furthest part of the globe which he can see? Ans. $13\frac{1}{8}$ ft.

11. Tangents are drawn to a circle (two inches in radius) at two points on the circumference 3 inches one from another; find with the protractor at what angle they will meet. Ans. 83°.

12. Draw two different circles to touch the same given line at the same given point. Such circles are said to touch one another. Draw figures for both cases, (1) when they touch on the same side of the line, and (2) on opposite sides of it.

13. Describe an equilateral triangle, side two inches, and by the means of it describe three circles, one inch in radius, each touching the other two.

14. To describe a circle to touch any two given lines.

Construction. Bisect the angle between the lines; take any point (A) on the bisector; drop a perpendicular from A on either of the given lines (*Third Construction*, Part I.); a circle from centre A with the length of the perpendicular as radius will be found to touch both lines.

15. A circle which touches all the sides of a figure is said to be *inscribed* in it.

To *inscribe* a circle in a given triangle.

Construction. Bisect two angles, and from the point of intersection of the bisectors (A) drop a perpendicular on one side (*Third Construction*); then a circle from centre A, with radius equal to the perpendicular, will touch all three sides.

16. Inscribe a circle in a square, whose side is two inches.

17. Inscribe an equal-sided octagon in a given circle.

18. Describe a square of 3 inches side, inscribe a circle in the square, and form an octagon by cutting off the corners with tangents to the circle.

19. Show by a figure that the tangents at the angular points of a triangle inscribed in a circle intersect the opposite sides produced in 3 points which are situated in a straight line.

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20. Inscribe any hexagon (with unequal sides) in a circle, and show by a figure that the intersections of the three pairs of opposite sides (viz. of the first and fourth, of the second and fifth, and of the third and sixth,) when produced to meet, lie on the same straight line.

21. Take any six points in the circumference of a circle, and draw tangents at them, thus forming a circumscribed hexagon: then join each pair of opposite angular points: the three lines joining the pairs of opposite angles will be found to pass through the same point.

22. Take any triangle inscribed in a circle, and from any point in the circumference of the circle drop perpendiculars on the sides of the triangle, or its sides produced (by *Third Construction*, Part I.); the feet of these three perpendiculars (that is, the points at which they meet the sides produced) will be in the same line.

23. Draw four intersecting lines so as to make four triangles, and draw circles about each of the triangles; these four circles will be found all to pass through the same point.

24. To draw a tangent touching both of two given circles, each of which is exterior to the other.

Construction. Take any two circles, with unequal radii. Let A be the centre of the greater circle, B of the less. Round A describe a third circle, with radius equal to the difference of the two circles; from B by (8, Part III.) draw a tangent to the new circle, touching it in C; join AC, and produce it to cut the outer circle in D; then at D erect a perpendicular to the radius, touching the larger circle; it will be found also to touch the other.

25. Take another pair of circles, and repeat the construction, with this difference that the radius of the third circle (centre A) is to be equal to the *sum* of the radii, instead of the *difference*. Another tangent common to both circles will be found.

26. Take two circles, as in (24), but with equal radii, and draw a common tangent to them.

27. Describe two circles with radii 1 and 2 inches, the centres being five inches apart, and describe four common tangents to them by (24) and (25).

28. Describe three circles ½ inch in radius, each of which touches the other two, and about them describe an equilateral triangle, each side of which touches two of the circles; measure the side of the triangle. Ans. 2¾ inches.

29. Describe two circles with radii 2 and 3 inches, the centres being five inches apart, and describe *three* common tangents to them.

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